Bayesian statistical tests for operational inverse modeling in response to a radiological atmospheric release

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Problem definition and objectives

Context

- In an event of an accidental release of radionuclides, authorities need to know (if possible in advance) the impacted area.
- Numerical models are used to forecast the radioactive plume. The performance of these tools are mainly forced by the knowledge of the source field.
- Data assimilation methods (such as inverse modelling) have shown, at least at an academic level, good skills to help in this matter.

Objectives

To propose data assimilation methods to help forecasting radionuclides plume

- simple enough to be implemented in an operational context (Which assumptions ? Which simplifications ?) and to be understood by operators
- but still efficient.

Outline

1. Sequential semi-automatic data assimilation system
2. Bayesian tests for the identification of the release site

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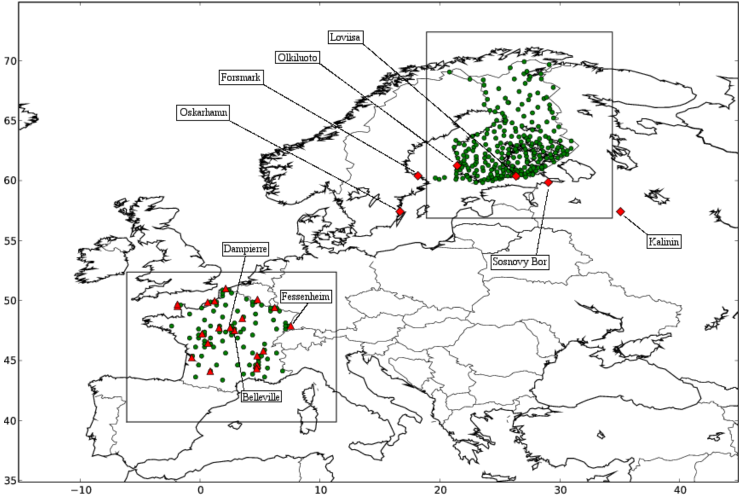
Framework of the study

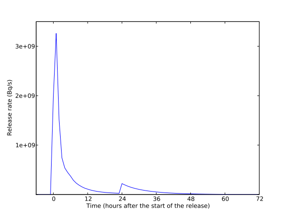


- France : 20 nuclear facilities - Monitoring network of 100 stations (Saunier et *al.* 2009)
- Finland : 6 sites - Monitoring network of 255 stations ("uljas" network)

Accidental dispersion synthetic experiment

Source term

- Hypothetical fast core meltdown, without hull breach (nuclear power plant)
- Dispersion of caesium 137.
- Intentional release 24 hours after the start of the accident → "double-peak" temporal profile.



Transport simulation and perturbed observations

- The networks are supposed to monitor the activity concentrations of ^{137}Cs (actually, most of them measure γ-dose).
- Transport simulated with Polair3D or SILAM → computation of synthetic observations $\mu^{\text{synth.}}$.
- Lognormal perturbations of synthetic observations :

$$\mu_i^{\text{perturb.}} \sim \exp\left(N(0,0.5)\right) \mu_i^{\text{synth.}} . \tag{1}$$

Inverse modelling scheme (1 / 3)

Cost Function

- Source-receptor relationship: $\mu = \mathbf{H}\sigma + \varepsilon$
- Assumption : the location of the accident is known $\rightarrow$ Unknown : temporal profile of the source σ (N_{imp} emission rates ; N_{imp} of the order of 10^2)
- Many more observations (though very noisy) than unknown
  $\rightarrow$ direct computation of the Jacobian matrix $\mathbf{H} \in \mathbb{R}^{N_{obs} \times N_{imp}}$ (column by column)
- Hypothesis on the errors : Gaussian following a normal distribution :

$$p(\varepsilon) = \frac{\exp\left(-\frac{1}{2}\varepsilon^{\mathrm{T}}\mathbf{R}^{-1}\varepsilon\right)}{\sqrt{(2\pi)^{N_{obs}}|\mathbf{R}|}} \tag{2}$$

- which leads to the following cost function :

$$\mathscr{L}(\sigma) = \frac{1}{2}\ln|\mathbf{R}| + \frac{1}{2}(\mathbf{H}\sigma - \mu)^{\mathrm{T}}\mathbf{R}^{-1}(\mathbf{H}\sigma - \mu) \tag{3}$$

- the term "$\ln|\mathbf{R}|$" can be important in the aim of on-line estimation of error covariance matrix (Dee, 1995).

Inverse modelling scheme (2 / 3)

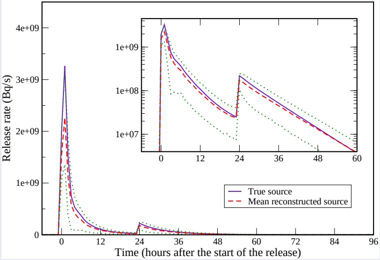
First try : R depends on the noisy observations μ

- $\mathbf{R} = \mathrm{diag}(\chi_1, \chi_2, \ldots, \chi_{N_{obs}})$ with $\sqrt{\chi_i} = r\mu_i$
- The estimated reconstructed source is then given by (BLUE) :

$$\overline{\sigma} = \left(\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}\mu \quad (4)$$

Results

- The scheme retrieves around 70% of the released mass.
- It can be proven that for a *very well observed* event, in the case of lognormal true errors, this scheme leads to a large underestimation of the source by a factor of $exp\left(-\frac{3}{2}\chi\right)$



Inverse modelling scheme (3 / 3)

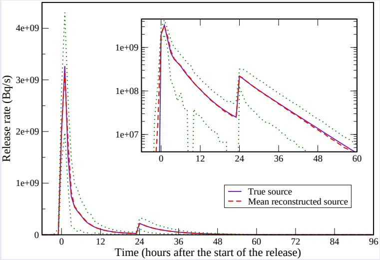
Improvement : $\mathbf{R}$ depends on the analysed measurements $\mathbf{H}\sigma$

- $\mathbf{R} = \mathrm{diag}(\chi_1, \chi_2, \ldots, \chi_{N_{obs}})$ with $\sqrt{\chi_i} = r\,[\mathbf{H}\sigma]_i$
- The cost function is then given by :

$$\mathscr{L}(\sigma) = \sum_{i=1}^{N_{obs}} \left(\ln\left([\mathbf{H}\sigma]_i\right) + \frac{1}{2r^2} \frac{\left([\mathbf{H}\sigma]_i - \mu_i\right)^2}{[\mathbf{H}\sigma]_i^2} \right) \qquad (5)$$

Results

- The scheme retrieves around 100% of the released mass.
- The main estimation errors occur in the vicinity of the peak
- But these errors are attenuated by the use of this cost function.



Data assimilation scheme

Reconstruction of the source (analysis)

- Measurements in the interval $[t_\mathrm{a} - \Delta t_\mathrm{a}, t_\mathrm{a}]$ are collected (those in $[t_0, t_\mathrm{a} - \Delta t_\mathrm{a}]$ were already available). This allows to build the measurement vectors up to t_a : μ_a.
- One prolongates the source-receptor matrix **H** at t_a, by prolongating or computing all the elementary solutions from $t_\mathrm{a} - \Delta t_\mathrm{a}$ to t_a.
- Then one computes an estimate of the source term $\overline{\sigma}_a$.

Forecast

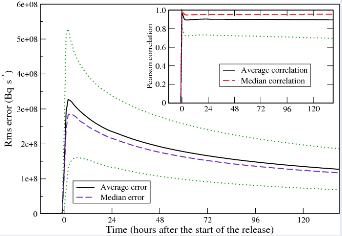
- A forecast is performed from t_a to t_f, using the transport model.
- Forecast driven by the best estimation of the source up to t_a ($\overline{\sigma}_a$), then by a guess of the source term from t_a to t_f (usually persistence hypothesis).

Results (1/2) : Statistical indicators

Reconstruction of the source

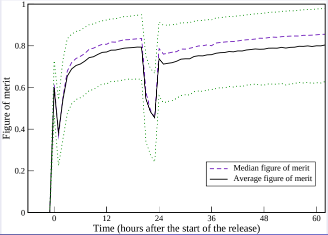
$$\text{rmse} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left([\overline{\sigma}]_i - [\sigma^t]_i\right)^2}$$

$$\rho = \sum_{i=1}^{N} \frac{[\overline{\sigma} - <\overline{\sigma}>]_i [\sigma^t - <\sigma^t>]_i}{\sqrt{\left(\sum_{j=1}^{N}[\overline{\sigma} - <\overline{\sigma}>]_j^2\right)\left(\sum_{j=1}^{N}[\sigma^t - <\sigma^t>]_j^2\right)}}$$

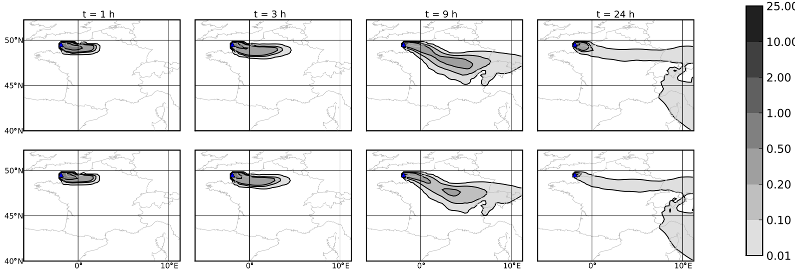


Forecast

$$\text{figure of merit} = \frac{\sum_{h\in S} \min([\overline{c}]_h, [c^t]_h)}{\sum_{h\in S} \max([\overline{c}]_h, [c^t]_h)}$$



Results (2/2) : Plume forecasts



- The source term is quickly (after 1-2 hours of observations) well-estimated.
- The forecast of the radioactive plume is of good quality.
- But in an operational context, the average behaviour of the system is not sufficient → One must pay attention to the cases where the system fails.

Fail cases : causes and solutions

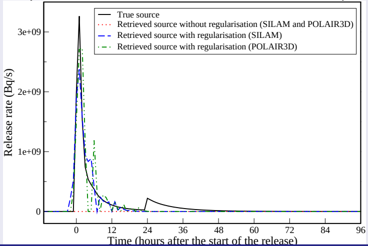
Analysis of the fail cases

Some power plants are located on the shores, near the frontiers or far from the monitoring networks → Some accidents are not well-observed → The inversion step is not achieved.

One alternative solution : Regularisation

Use of a background term : for example a Gaussian assumption for the source term distribution, with $\mathbf{B} = m^2\mathbf{I}$ the background error covariance matrix, leads to a new cost function :

$$\mathscr{L}(\sigma) = \sum_{i=1}^{N_{obs}} \left(\ln\left([\mathbf{H}\sigma]_i\right) + \frac{1}{2r^2} \frac{\left([\mathbf{H}\sigma]_i - \mu_i\right)^2}{[\mathbf{H}\sigma]_i^2} \right) + \frac{1}{2} \sum_{i=1}^{N_{imp}} \frac{\sigma_i^2}{m^2} \quad (6)$$



Outline

Objectives and general principles

If the release site is unknown ?

The operational data assimilation system needs to be complemented

- with more sophisticated methods that do not assume that the release localisation is known (parametrical or non-parametrical methods, progressive reduction of candidates group)
- or with statistical tools which indicate the probability of a power plant to be responsible for the accident, knowing the measurements.

Bayesian tests

Such statistical tests are based on Bayesian inference theory

$$p(\mu) = \int p(\sigma)p(\mu|\sigma)\,\mathrm{d}\sigma\,, \tag{7}$$

and differ from each other by the assumptions made on the source prior $p(\sigma)$

Gaussian prior (1/2)

Principles

If $p(\sigma)$ follows a Gaussian multivariate distribution, one obtains

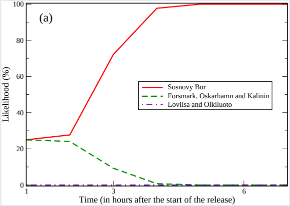
$$p_i(\mu) = \frac{\exp(-\frac{1}{2}\mu^{\mathrm{T}}\left(\mathbf{H}_i\mathbf{B}\mathbf{H}_i^{\mathrm{T}} + \mathbf{R}\right)^{-1}\mu)}{\left|\mathbf{H}_i\mathbf{B}\mathbf{H}_i^{\mathrm{T}} + \mathbf{R}\right|^{\frac{1}{2}}} \tag{8}$$

$p_i(\mu)$ represents the likelihood of the dataset μ provided the source prior statistics are Gaussian, and that the source is located at site i. $\mathbf{H}_i$ being the Jacobian matrix of site i (a submatrix of $\mathbf{H}$).

Gaussian prior (2/2)

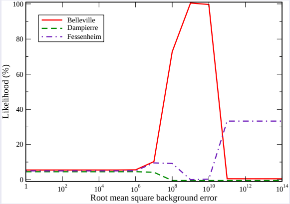
Results

- The (hypothetical) accident occurs in Sosnovy Bor power plant
- After 3 hours, the likelihood is about 75% for this power plant
- After 5 hours, 100%



Range of validity

- This indicator strongly depends on **B**.
- But in this case, there is a range of validity of four orders of magnitude.



Non-Gaussian prior (1/2)

Principles

- Main assumption : The temporal profile σ_b of the source rates is known, but not their real magnitude.
- Thus, the source is assumed to be of the form $\sigma = \lambda \sigma_b$
- Different prior $p(\lambda)$ are assumed (gamma distribution, semi-gaussian) leading to different estimations of $p_i(\mu)$.

Example : with a semi-gaussian prior $p(\lambda) = \sqrt{\frac{2}{\pi}} e^{-\frac{\lambda^2}{2\theta}}$

$$p_i(\mu) = \frac{e^{\frac{(\mu^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}_i\sigma_b)^2}{2(\theta^{-1}+\sigma_b^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}_i\sigma_b)}}}{\sqrt{\theta^{-1}+\sigma_b^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}_i\sigma_b}} \times \left[1+\Phi\left(\frac{\mu^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}_i\sigma_b}{\sqrt{2(\theta^{-1}+\sigma_b^{\mathrm{T}}\mathbf{H}_i^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{H}_i\sigma_b)}}\right)\right] \tag{9}$$

where $\Phi(u) = \frac{2}{\sqrt{\pi}} \int_0^u e^{-x^2} dx$ is the error function.

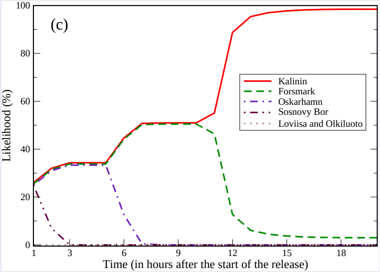
Non-Gaussian prior (2/2)

Global performance

- When σ_b is close from the true source, average results are excellent (2-3 hours to obtain at least 80% for the correct site).
- When σ_b is different from the true source (wrong shape, wrong magnitude), the average performance still reaches 80%, but it does so later (typically 5 hours later).

Critical example

- Fictitious accident in Kalinin power plant, with a poorly observed plume.
- The Gaussian test failed.
- The reconstruction of the source, knowing the release site was difficult (need of regularisation).
- The non-Gaussian tests managed to gradually identify the responsible site.



Conclusion

A semi-automatic data assimilation system, using inverse modelling techniques, has been proposed to forecast a radioactive plume in an event of an accidental release from a nuclear power plant.

- Very good average performances (source quickly well-estimated, plume accurately forecasted).
- Fail situations have been identified and some complementary solutions have been proposed (regularisation, international network).

In the case where the release site is unknown, statistical tests have been proposed and implemented to help identifying the responsible site.

- Excellent results in average and even good results in critical situations.

These methods are designed for operational context (simple, fast, but still efficient) and we hope that they will be implemented by agencies, for example IRSN.

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