Estimation of Friction Parameters and Laws in 2D Shallow-Water Gravity Currents on the f-Plane, by Data Assimilation

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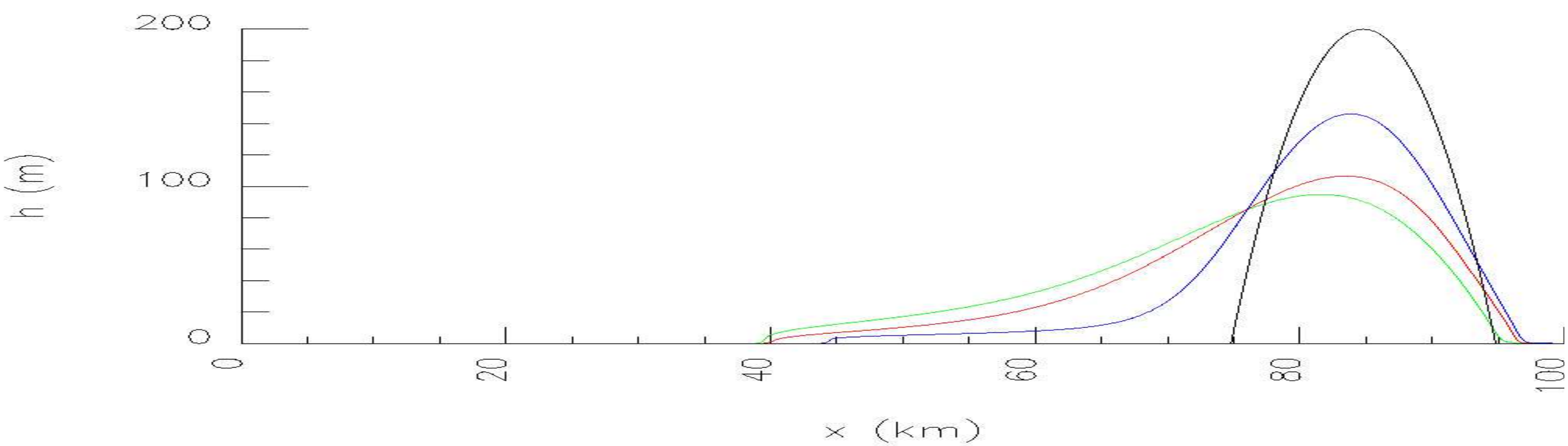
Etudes et paramétrisation
des flux turbulents pour les
courants gravitaires océaniques
par assimilation de données



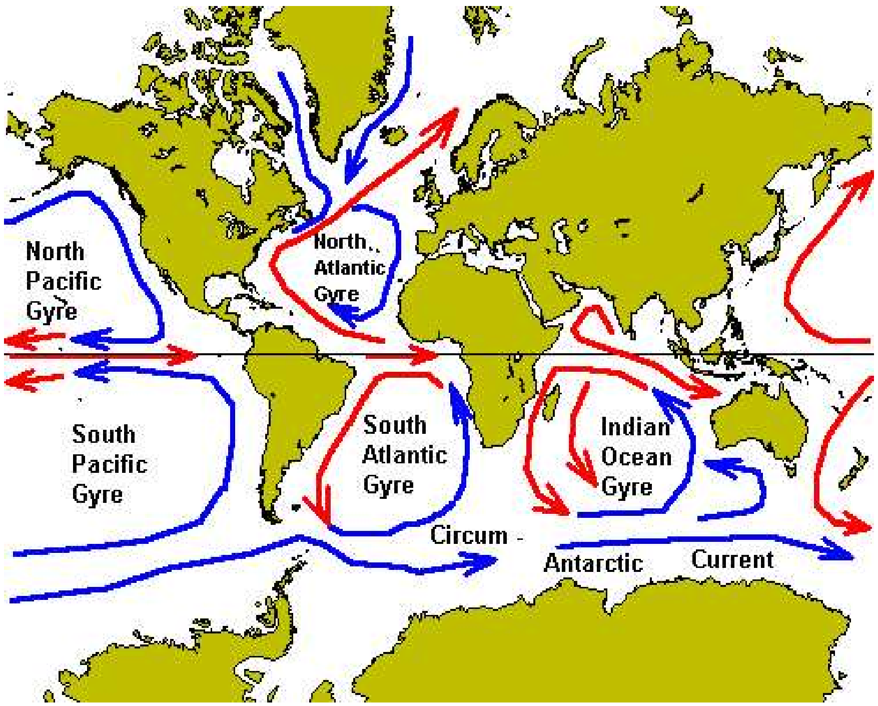
Ocean Circulation

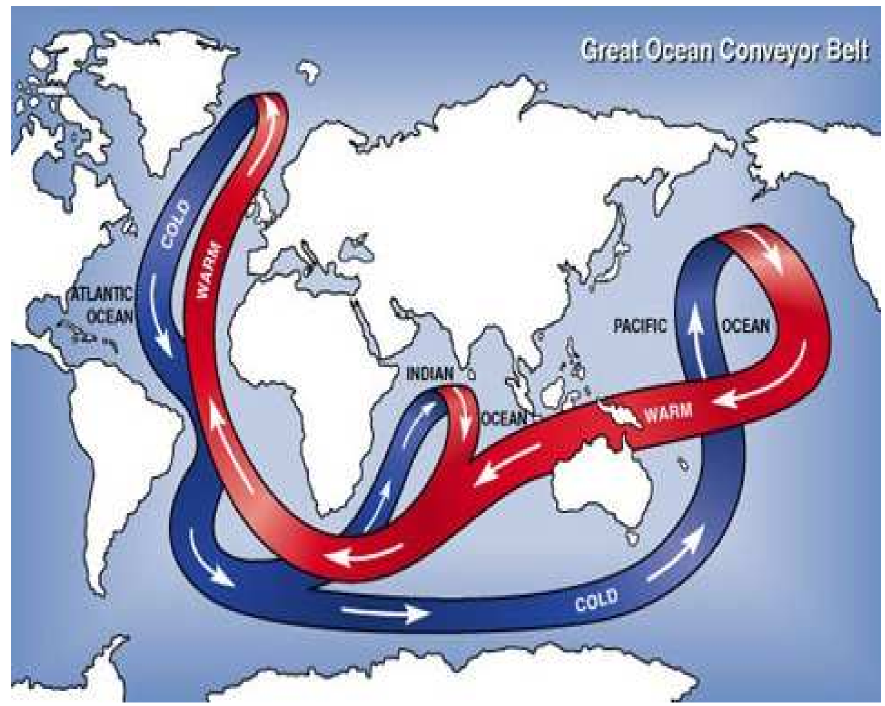
Gyre

"weather"

Overturning

"climate"





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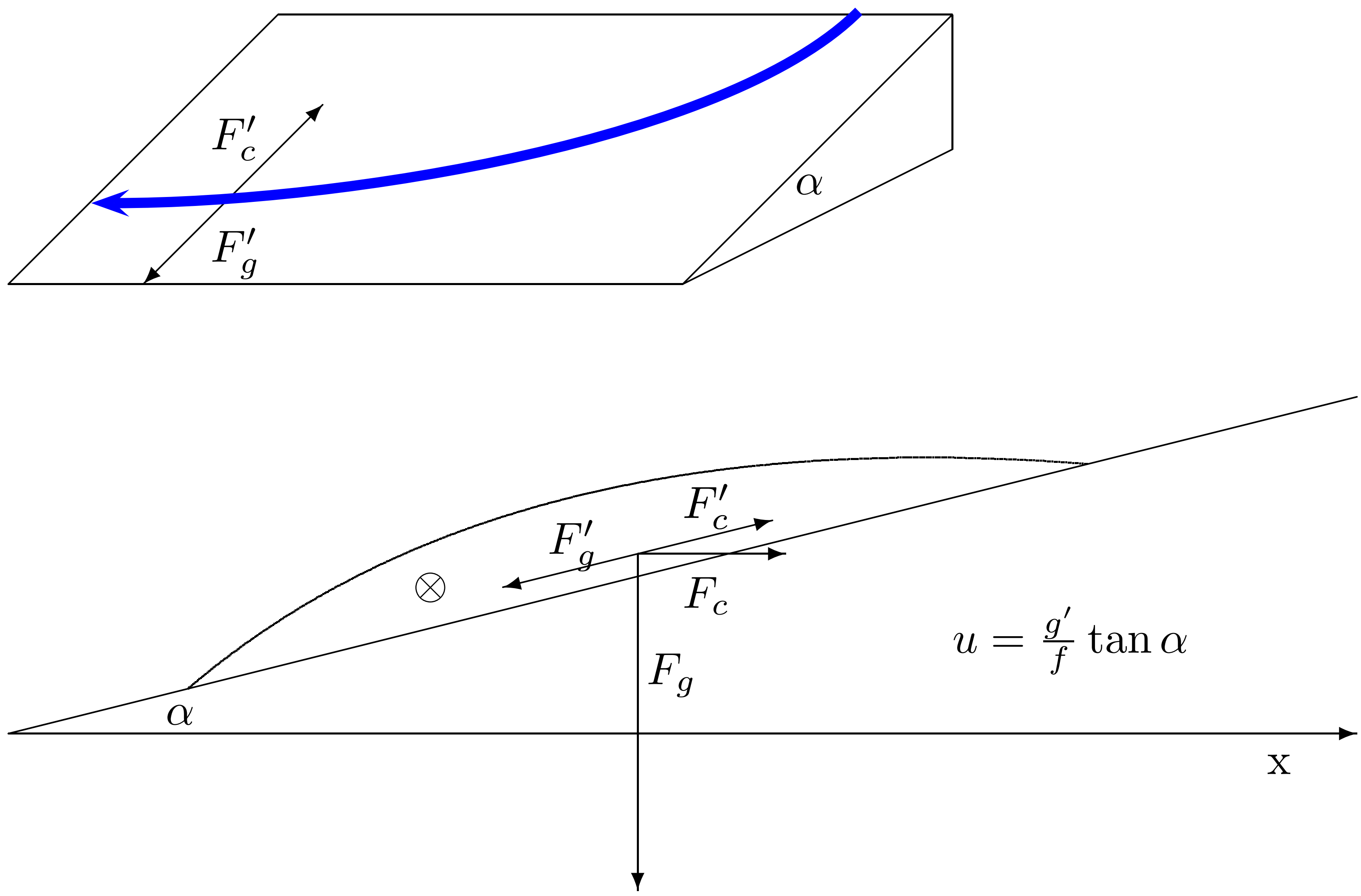
Gravity Currents in a Rotating Frame

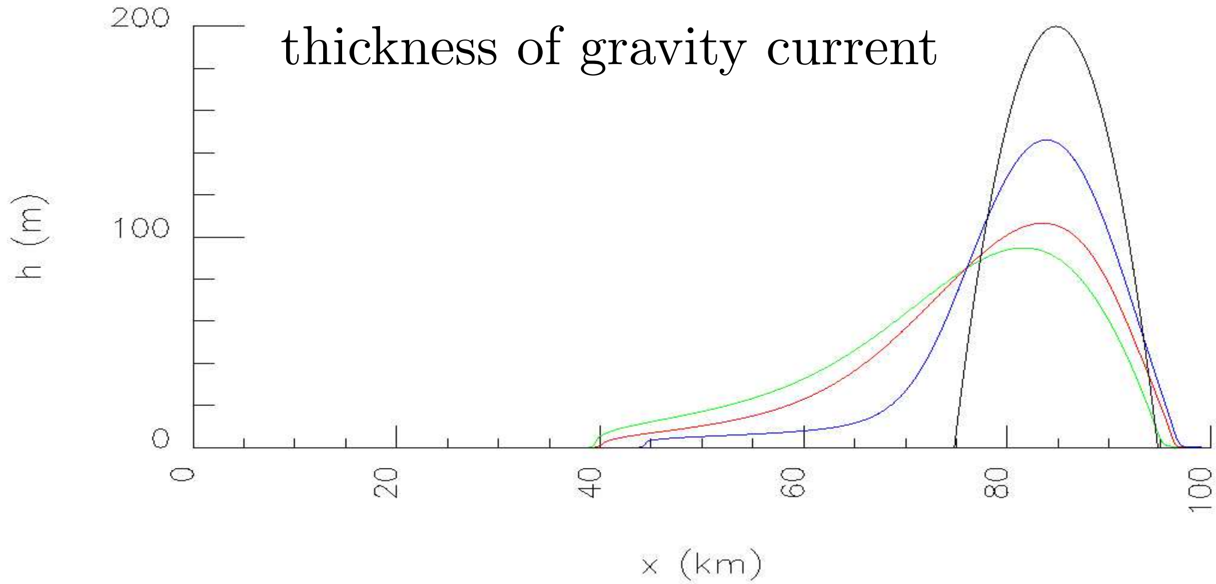


What Makes Gravity Currents Flow Down-Slope ?

- To first order gravity currents change neither depth nor speed (geostrophic equilibrium).
- Dissipative turbulent fluxes of momentum and density perturb the geostrophic equilibrium (friction, mixing, entrainment, detrainment).
- These turbulent fluxes can NOT be explicitly resolved in today's (and tomorrows) models of the global ocean circulation.
- We have to find efficient parametrisations and the correct parameter values

Time Evolution of a Cross Section

$$
\begin{aligned}
\partial_t u + u\partial_x u - \quad & f v + g'(\partial_x h + \tan\alpha) && = -Du + \nu\partial_x^2 u, \\
\partial_t v + u\partial_x v + \quad & f u && = -Dv + \nu\partial_x^2 v, \\
\partial_t h + u\partial_x h + \quad & h\partial_x u && = \nu\partial_x^2 h.
\end{aligned}
$$

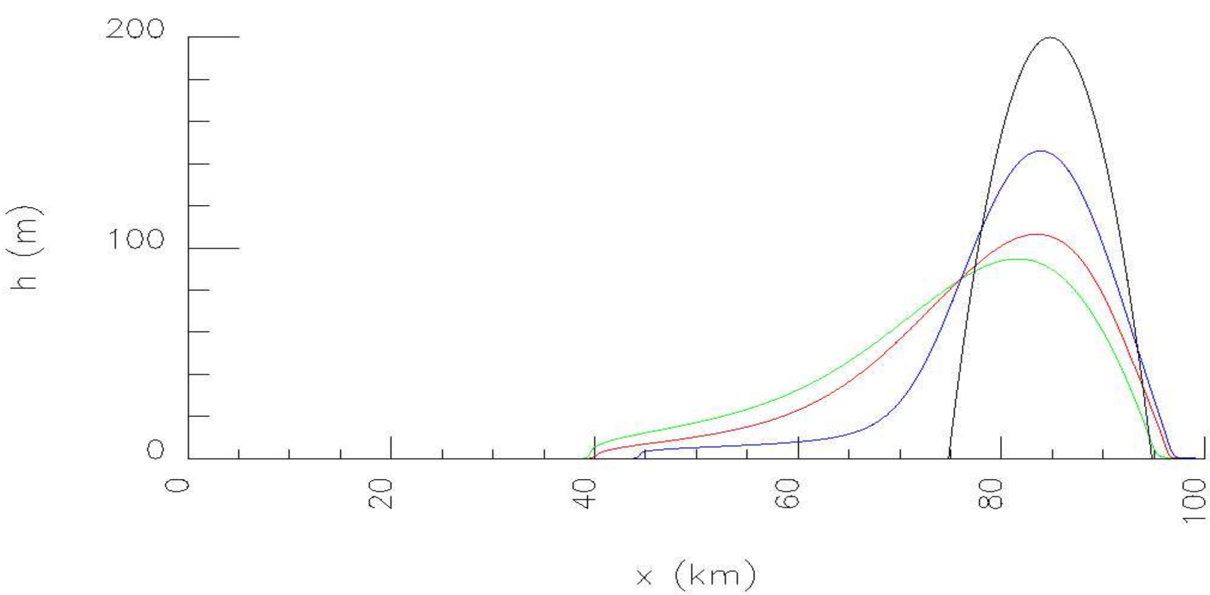


The Friction

$$D = D(x,t) = \frac{1}{h}(\tau + c_D|\mathbf{u}|)$$

Two friction laws:

- Linear Rayleigh friction (τ)
- Quadratic drag law (c_D)



Why use Data Asimilation?

"I am the wisest man alive, for I know one thing, and that is that I know nothing" (Socrates).

I do not know ...

- the roughness of the ocean floor.
- the difference between roughness and obstacles
- the effects of roughness change.
- the roughness type of the ocean floor "k" vrs. "d".
- the direction of roughness elements.
- the sediment suspension.
- tidal and wave currents (short time-scale)
-

If I knew all this: "The matter is far from beeing understood" Jiménez, Ann. Rev. Fluid Mech. (2004).

The Estimation Procedure

Augmented state vector (containing the parameters):

$$\mathbf{x}(x,t) = (h(x,t), u(x,t), v(x,t), \tau, c_D) \tag{1}$$

The tool: EnKF (Ensemble Kalman Filter) which updates the ensemble mean and variance by updating each ensemble member:

$$\mathbf{x}_i^a = \mathbf{x_i^f} + \mathbf{K}\left(h^{obs} + \epsilon_i - \mathbf{H}\mathbf{x^f}_i\right) \tag{2}$$

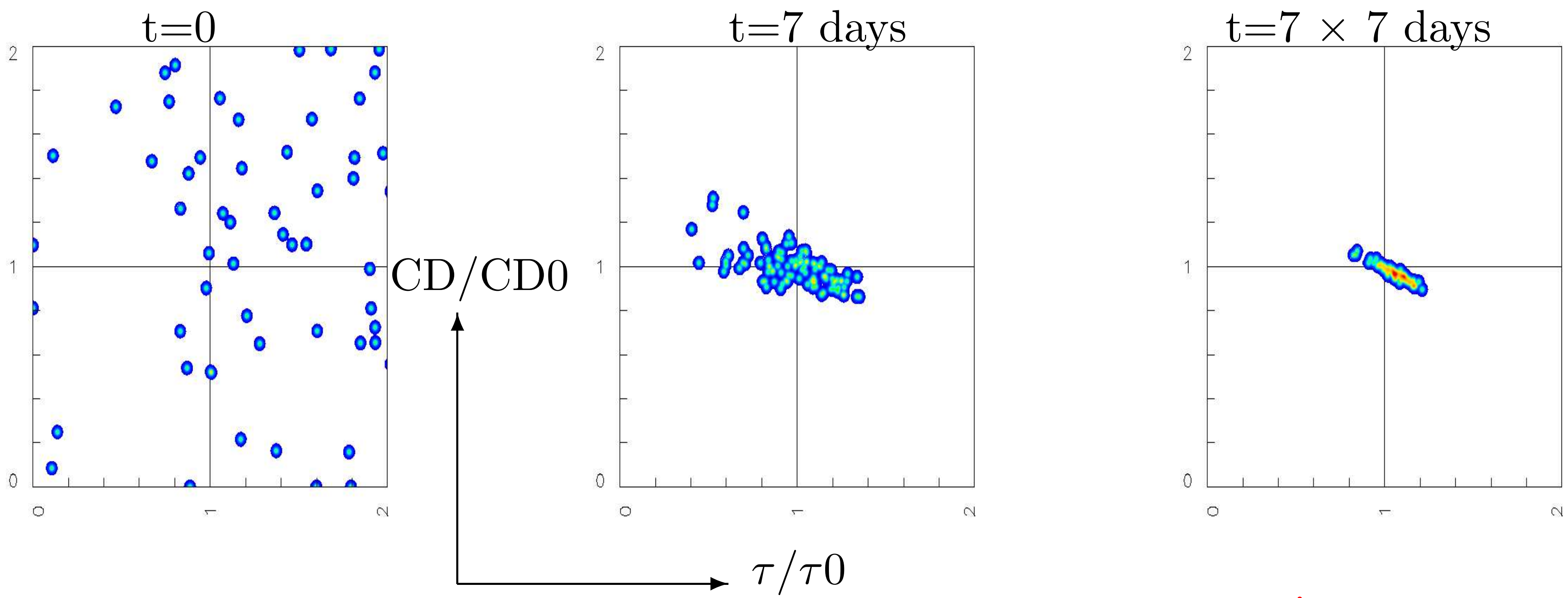
$$\mathbf{K} = \mathbf{P}\mathbf{H^T}(\mathbf{H}\mathbf{P}\mathbf{H^T} + \mathbf{W})^{-1} \tag{3}$$

$$\mathbf{P} = \frac{1}{m-1}\sum_{i=1}^{m}(\mathbf{x_i^f} - \langle \mathbf{x^f} \rangle)(\mathbf{x_i^f} - \langle \mathbf{x^f} \rangle)^T, \tag{4}$$

Choice of initial distribution of parameter values?

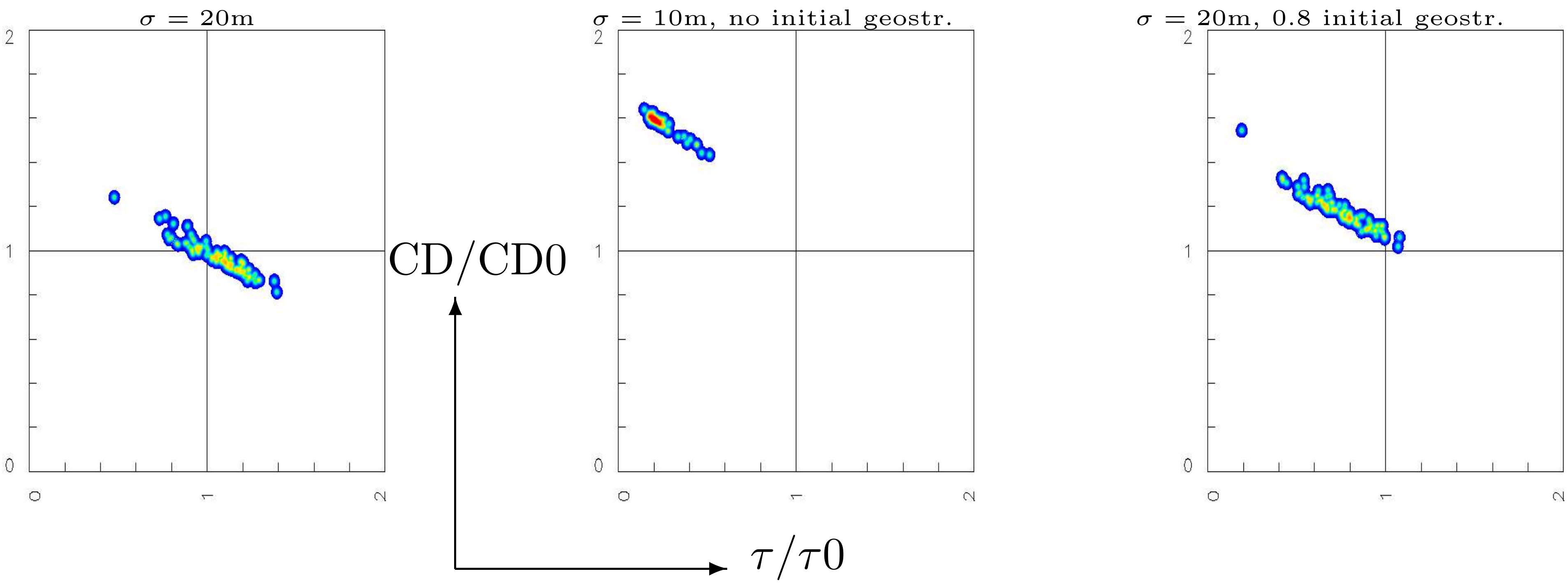
Can we obtain r and c_D from ONLY observing $h(x)$?

Assimilation every hour, with an observation error of $10m$,
$\tau_0 = 5.10^{-4}$ ms^{-1}, $c_{D0} = 5.10^{-3}$



Yes!

Why is the ensemble aligned?



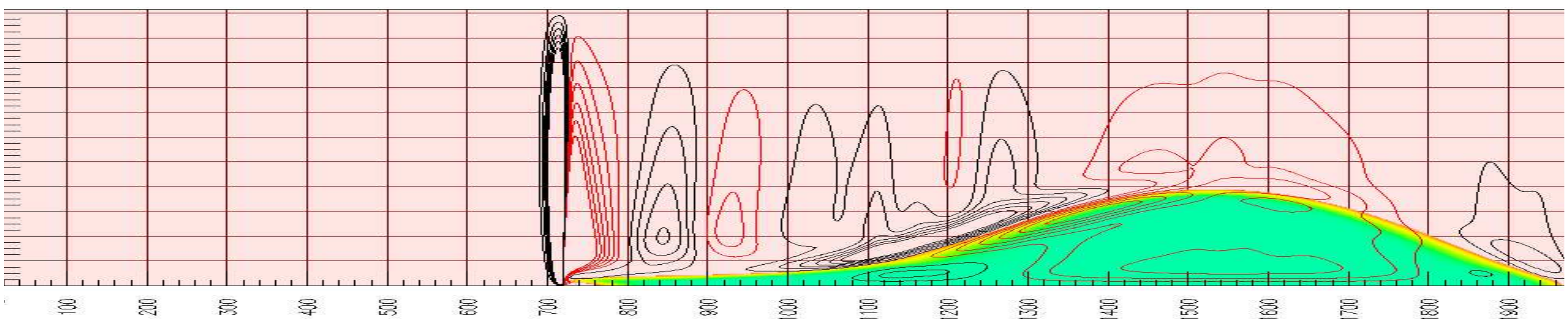
Because it is easy to estimate the average friction $\langle D \rangle_{x,t}$, where:

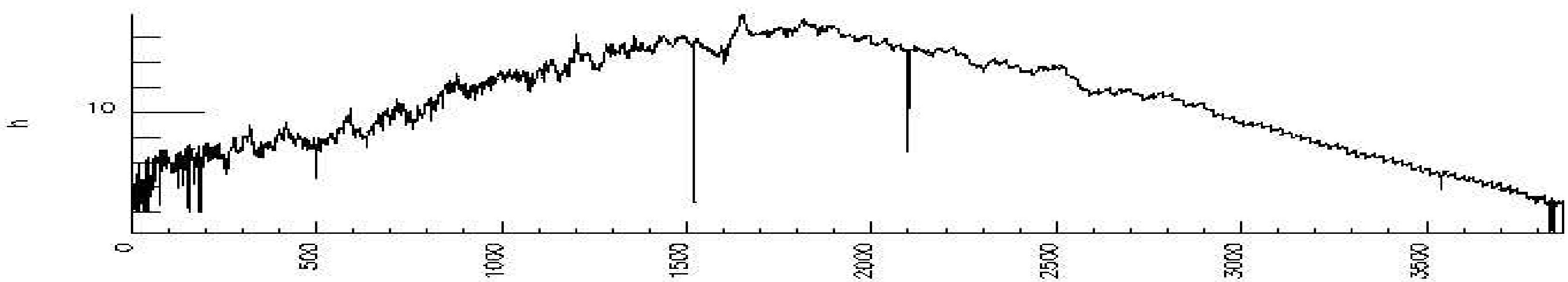
$$D = (\tau + c_D |\mathbf{u}|)$$

but it is more difficult to separate it into linear Rayleigh friction vs. quadratic drag law. We have a fast projection on the "slow manifold" corresponding to $\langle D \rangle_{x,t} = const.$ followed by a slow convergence within the "slow manifold."

Next:

- Move from twin experiments to connecting models in a “lowrarchy”.
- Connecting models to experiments.
- Estimate parameters and determine laws.





Conclusions:

- The EnKF is a robust tool for estimating parameters
- Parameter estimation can be used to determine laws (by estimating parameters).
- The spread of the ensemble tells us something about the physics of the problem.
- The here presented results are likely to be general for parameter estimation.